Rapid mixing of the flip chain over Non-Crossing Spanning Trees

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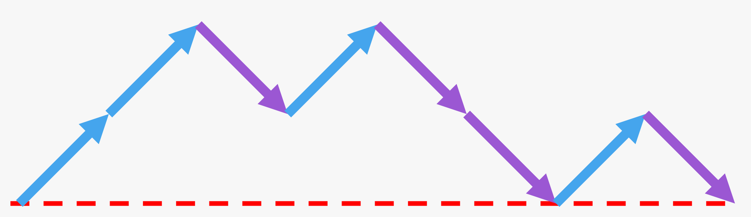
1-Catalan structures



1-Dyck path of length 2n

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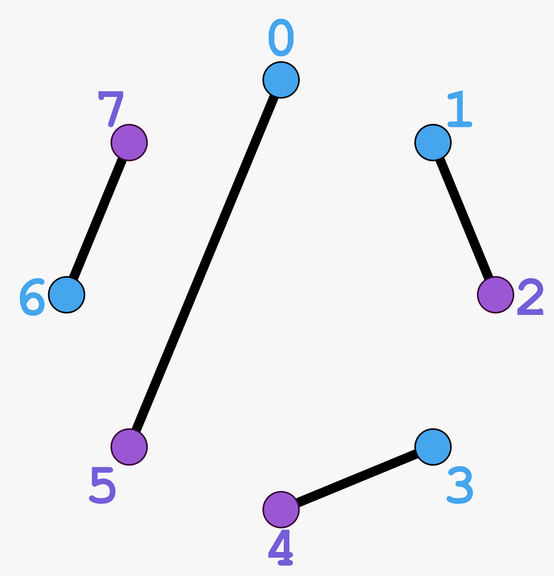
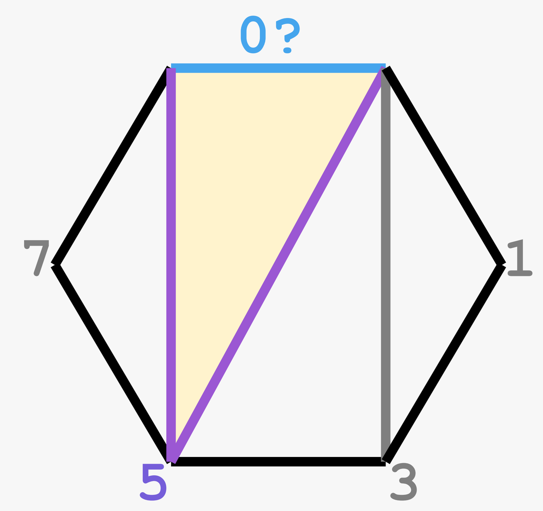
Bracket sequence of length 2n



Non-crossing perfect matching between 2n points



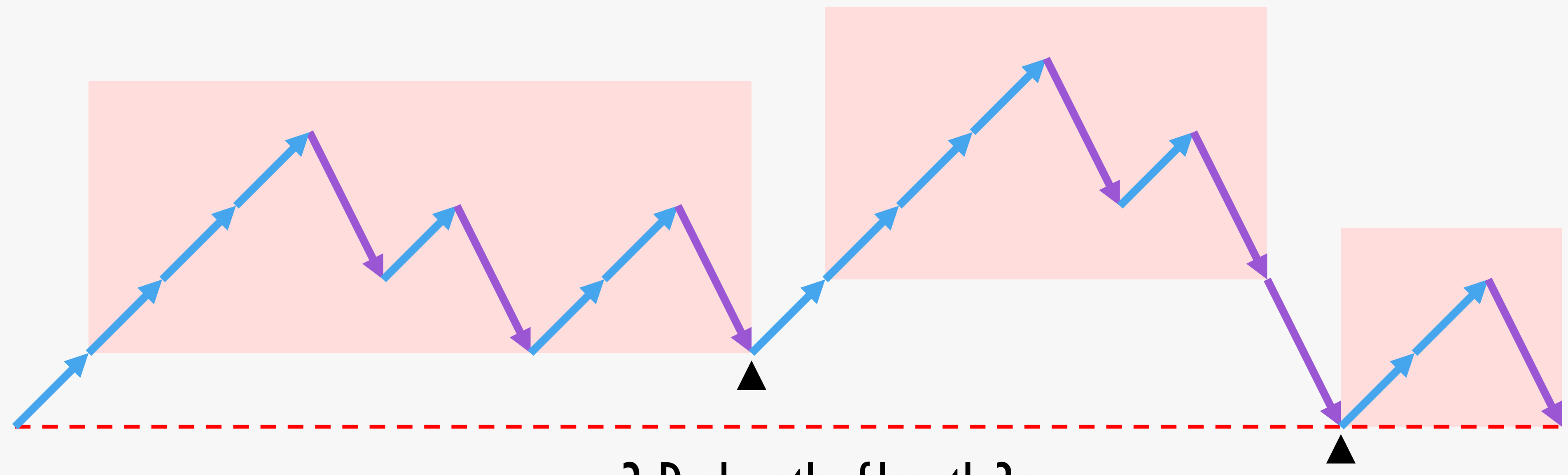
Triangulation of n+2 gon

$$C_n = \sum_{i=1}^{n} C_{i-1} C_{n-i}$$

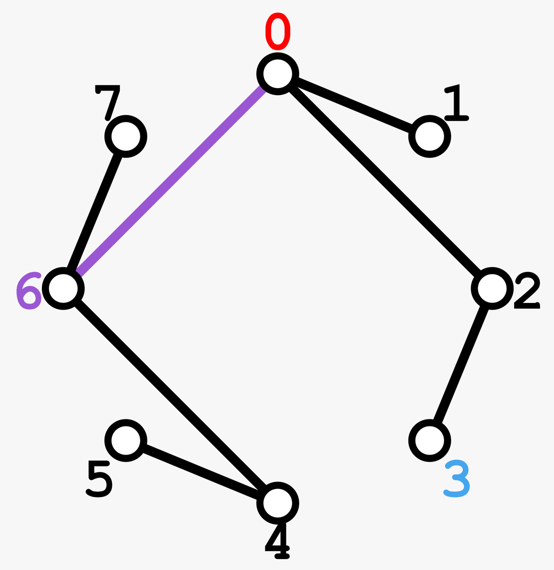
How you prove recurrences

bijections between these structures

2-Catalan structures

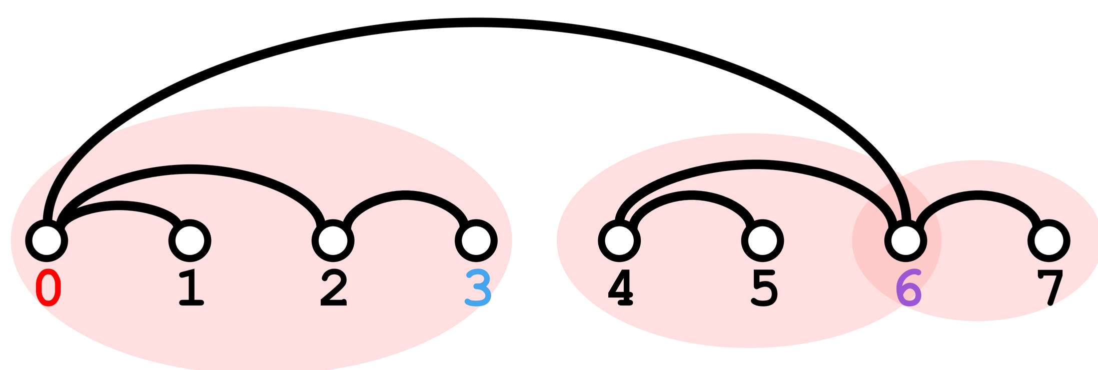


2-Dyck path of length 3n

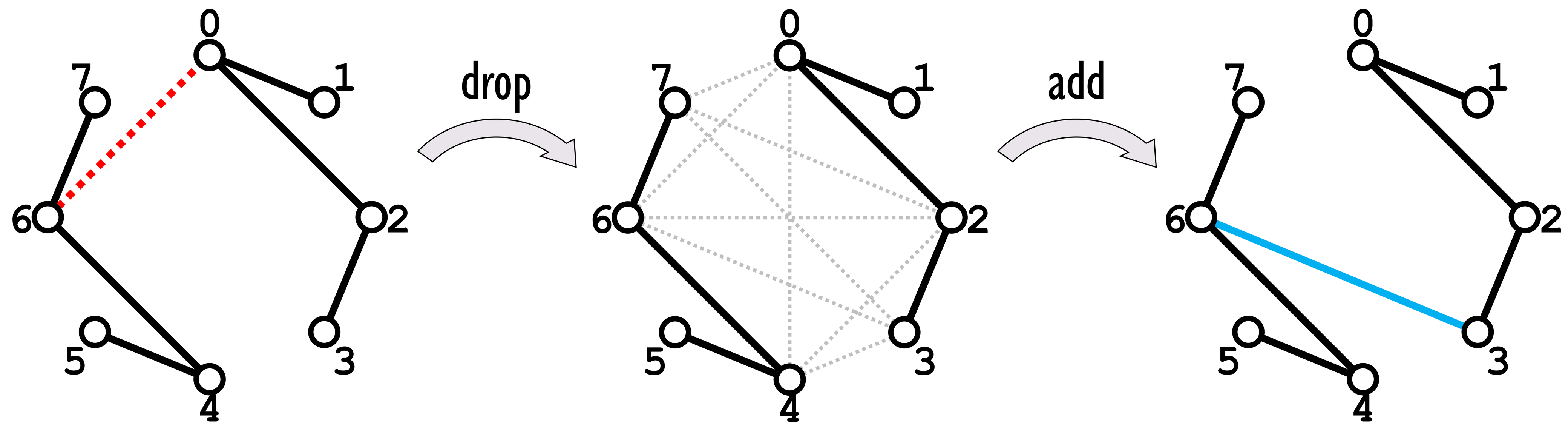


Non-crossing spanning trees of n+1 points

Alternatively,



$$C_n^{(2)} = \sum_{\substack{i,j,k \in \mathbb{Z}_{\geq 0} \\ i+j+k=n-1}} C_i^{(2)} C_j^{(2)} C_k^{(2)}$$

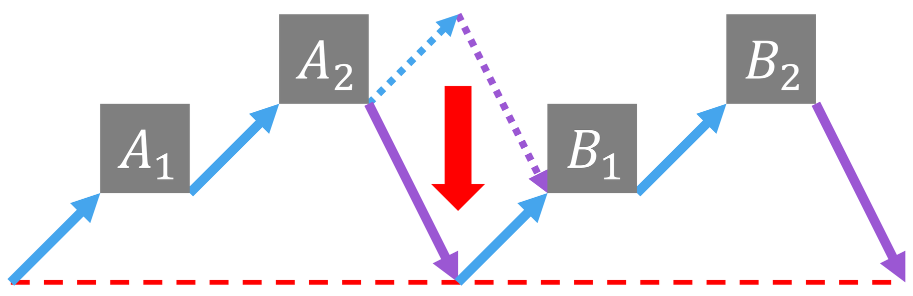


Mixing time (speed of convergence) of this chain?

Mixing times of Catalan chains

Catalan matroid down-up walk	$\Theta(n \log n)$ [Cryan-Guo-Mousa, 2019]	
Lattice (k-Dyck path) adjacent move	$\Theta(n^3 \log n)$ [Wilson, 2004]	
$\widetilde{\Theta}(n^{1.5})$? Triangulation flip chain	$\widetilde{\Omega}(n^{1.5})$ [Molloy-Reed-Steiger, 1997]	$\widetilde{O}(n^2)$ [Alev-Frishberg-Sarantis-Tetali, 2026]
$\widetilde{\Theta}(n^2)$? NC perfect matchings swap chain	$\widetilde{\Omega}(n)$	$\widetilde{O}(n^4)$ [Cohen, 2016]
$\widetilde{\Theta}(n^{1.25})$? NC spanning trees flip chain	$\widetilde{\Omega}(n)$	$\widetilde{O}(n^8)$ [This work]

Canonical path: NCST flip mixing = 2-DP adjacent move mixing + efficient mapping between moves (evaluated by the number of preimages)



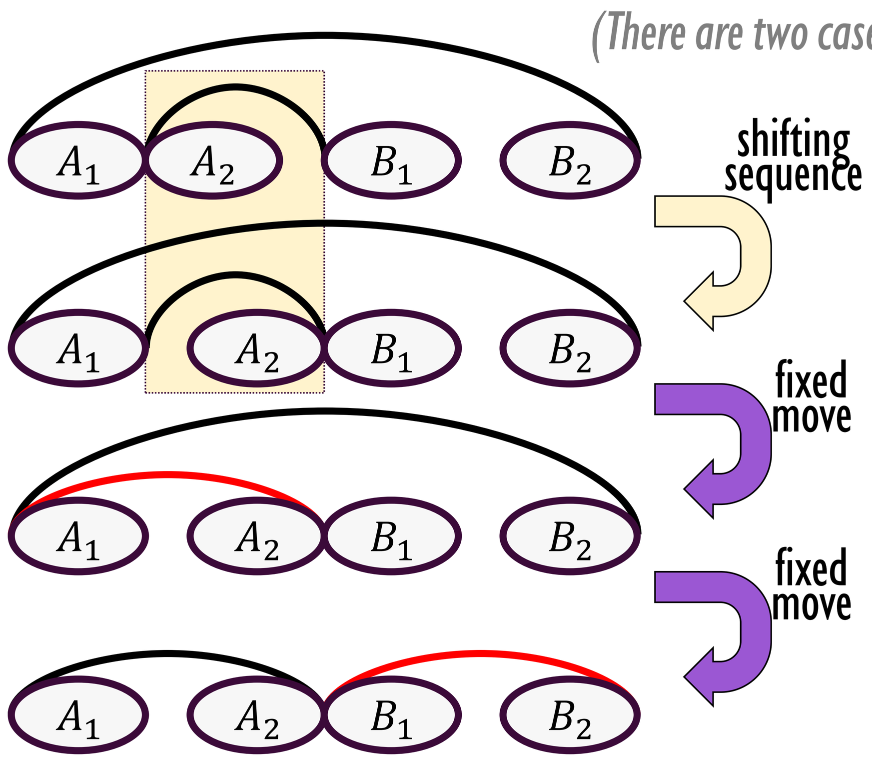
(There are two cases)

Extra info: type of the move & recursion level

#preimages ≤ #extra info = 12n

shifting: moving the only connected component under an arc

unique move/recursion sequence: recover initial and final states easily



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